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## Monterey, California



A PROGRAM TO COMPUTE MAGNETIC ANOMALY  
DETECTION PROBABILITIES

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The program in this report is presented without representation or warranty of any kind.



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## I. Introduction

This report contains user instructions, a listing and documentation for a microcomputer BASIC program that can be used to compute an estimate of the probability that a magnetic anomaly detection (MAD) system such as the AN/ASQ-81 will detect a submarine during an encounter.

The program generates detection probabilities based on two encounter models. In the first encounter model, the detection system uses a crosscorrelation detector. In the second encounter model, the detection system uses a square law detector. Relative to operationally realizable values, probabilities based on the first model represent upper bounds and those based on the second model represent lower bounds. For both encounter models, a magnetometer signal is proportional to the magnitude of the projection of an anomaly field on the earth field, a submarine anomaly field is a dipole field and the earth field and the noise do not change with changes in the position of the magnetometer. Also, for both encounter models, an encounter is a straight line encounter with constant vertical separation.

The encounter models can be interpreted as models of a magnetic anomaly detection system on an aircraft that is moving with constant course, speed and altitude in an encounter with a submarine moving with constant course, speed and depth. Or, they can be interpreted as models of a stationary magnetic anomaly detection system in an encounter with a submarine moving with constant course, speed and depth.

The program parameters include encounter latitude and longitude, submarine induced magnetic moments, submarine permanent magnetic moments, submarine course, speed and altitude, magnetometer course, speed and depth, encounter lateral range (the horizontal range at the closest point of approach in a straight line encounter) and false alarm rate. (In the program, a false alarm is the event that the detection system classifies noise as a dipole signal.)

## II. Program User Instructions

As listed in Appendix 10, the program can be run under a BASIC language that is compatible with IBM PC BASIC. If the listing is used to enter the program through the keyboard, then the program should be saved with the name MAD.BAS or the value of N\$ on line 20 should be changed to the file name under which it is saved.

The program contains user instructions in the form of query and parameter limitation messages. As an example of the former, after starting the program under BASIC, the following message should appear:

Magnetic Anomaly Detection (MAD) Lateral Range Function

generate or print a program data file (g/p)?

By entering p, data can be printed from a program data file that was generated by the program. By entering g, a program data file can be generated for a set of user specified conditions. With either response, a sequence of additional queries is displayed. These queries require either an indicated response or a parameter value as the input. If the initial response is g, the sequence includes queries whose responses determine whether or not an auxiliary data file will be generated that can be used for future input of magnetic, processing or kinematic data. In particular, the first query in the sequence gives the option of using a combined magnetic, processing and kinematic data file. If the response indicates that it should

be used and the file is available, the parameter values that remain to be entered in order to generate a program data file are the following: the false alarm rate, the magnetic noise, the maximum encounter lateral range and the lateral range step. The combined file should be used only if the effect of varying just one or more of these parameters is desired. If the response indicates that the file will not be used, queries concerning magnetic, kinematic and processing parameter values are displayed.

After all of the program parameter values have been entered, a query is displayed giving the option of generating the combined file. Then a query giving the option of generating a program data file, a query giving the option of printing the encounter parameter values and a query giving the option of printing lateral range function values are displayed. The lateral range function values are the encounter detection probabilities indexed by lateral range. The parameters maximum lateral range and lateral range step determine the index lateral ranges of the encounters for which probabilities of detection are computed.

The program generates magnetic signal values that correspond to points in time during an encounter. Following the lateral range function query, a query is displayed that gives the option of printing the magnetic signal values for an encounter. If the option is exercised, the option is repeated. When the option is not exercised, a query is displayed that gives the option of

generating or printing a new program data file. If this option is not exercised, the program ends.

Some suggested guides for determining parameter values can be found in Section IV of this report.

### III. Encounter Model Limitations

In the two encounter models, noise is determined by a gaussian process. This may be a limitation in describing a stationary magnetometer. In this case, the noise must account for geomagnetic, instrument and magnetohydrodynamic noise. And it may be a significant limitation if the magnetometer is moving. In this case, in addition to the above sources, the noise must account for maneuver noise and geologic noise.

The signal is that of a magnetic dipole that moves relative to the magnetometer. Describing a ship magnetic anomaly as a dipole anomaly should not be a significant limitation for encounter slant range at the closest point of approach (CPA) that is greater than one hull length. The detection decision is based on samples from a single time interval (window) that is centered on the CPA. The length of the sampling interval and the sampling rate are parameter values that are inputs to the program. In terms of signal-to-noise ratio, there is an optimum sampling interval length (integration time) and sample rate. Although dipole signal energy is not symmetrically distributed about the CPA time, for a given sampling interval, the difference between the signal energy for the optimum interval location and the CPA centered location should not be significant in most cases.

The encounter model magnetometer noise samples are values of independent identically distributed random variables. The standard deviation of these random variables is referred to as



the magnetic noise and the variance is the noise in the sense of the signal-to-noise ratio. The magnetic noise of the encounter models is determined by a gaussian random process. The degree of correspondence between this process and operational noise depends on the nature of the dominant operational noise sources and on the magnetometer input filtering (noise whitening).

In the encounter models, the intervals are adjacent but not overlapping and a detection statistic corresponds to each sample interval and its value is determined by the sample values. If the value of the detection statistic for an interval equals or exceeds a threshold value, a detection is indicated. The threshold value is determined by the false alarm probability which in turn is determined by the false alarm rate and the sampling interval length.

A detector that used a moving sample interval that was generated by replacing the oldest sample by the newest one would correspond more closely to the detector in an operational detection system. In a model of the detector, since the sample windows overlap, the detection statistics would represent a sequence of random variables that were correlated over an interval equal to the width of the sample interval. Because of this dependence, it seems unlikely that the results that would be obtained with an encounter model based on an overlapping interval detector would differ significantly from those obtained with an encounter model based on a nonoverlapping interval detector.

For encounter lengths of the order of a few nautical miles or less, the straight line encounter condition should not be a significant limitation. In particular, this should be the case for a fixed magnetometer since, for a submarine (or surface ship) target, vertical separation and course changes should be less likely to occur.

Other models are available that can be used as the basis for computing an estimate of the probability that a magnetic anomaly detection system will detect a submarine during an encounter. For example, one that is described in Appendix 8 can be used to determine the slant ranges of straight line encounters for which the detection probability is equal to a specified value. The parameter values that are required to do this are an average submarine dipole moment, a detection system capability factor and a noise factor. Values for these parameters can be determined from operational data. However, the values are specific to averages over a particular set of encounter conditions. An advantage of the two encounter models relative to this model is their adaptability to different magnetic, processing and kinematic conditions.



#### IV. Parameter Values

The magnetic parameter value queries are generally explanatory with regard to the value that should be entered. This is also true of the kinematic parameter value queries. However, there is some ambiguity with respect the processing parameter value queries and the noise parameter value query. To reduce this ambiguity, a brief discussion of the common characteristics of the two encounter model processing parameters and the noise parameter is given below. This is followed by some guidelines for choosing these parameter values.

In both encounter models, a decision is made at the end of each sampling interval. The decision is either noise energy was present during the interval or noise energy plus signal energy was present during the interval. The sample intervals are adjacent, equal width, nonoverlapping time intervals. The number of samples that are input in a sample interval is determined by the sampling rate and the interval length.

The program default choice for the sampling rate is  $2 \cdot \text{MAXF}$  where MAXF is a parameter that is labeled the maximum magnetic signal frequency. This sample rate is the Nyquist rate for an ideal low pass filter. However, the signal in a sample interval that is computed by the program represents an unfiltered dipole signal. This is a reasonable approximation if the signal energy that is associated with signal components greater than MAXF is relatively small. As discussed below, the noise energy in a sample interval should be considered to be proportional to MAXF

in order to be consistent with the encounter models. Ideally, a default choice for MAXF would make the ratio of the signal energy to the noise energy a maximum for the sampling interval of an encounter. The program default choice for MAXF is  $2 \cdot \text{MAXVM} / \text{MINR0}$  where MAXVM is a user estimated maximum encounter relative speed converted from knots to meters per second and MINR0 is a user estimated maximum slant range at CPA in meters in terms of a just detectable target. (A precise definition of a just detectable target can be made in terms of a specified detection probability, false alarm probability and target dipole moment.) The default choice for MAXF is consistent with the observation in Reference 1 that if an optimum value for MAXF is determined for a minimum dipole moment target, then no significant increase in MAXF is required in order to maintain a required detection probability if the encounter lateral range is decreased even though the signal energy spectrum is shifted to higher frequencies.

Because of the detection statistics and the gaussian noise model, if there were no penalty for decision delay, a sample interval length for a signal should be chosen equal to the signal duration, since this would make the detection probability for an encounter a maximum. In the program, a default choice for the sample interval length (integration time) is  $2 \cdot \text{MAXR0} / \text{MINVM}$  where MINVM is a user estimated maximum encounter relative speed converted from knots to meters per second and MAXR0 is a user estimated maximum slant range at CPA in meters for a detectable encounter (in terms of a specific detection probability and false

alarm probability) with a user specified maximum dipole moment target. This choice might be considered a balance between minimizing decision delay and maximizing detection probability. In the encounter models, the sample intervals are located so that the CPA time is at the center of a sample interval and this sample interval is the only one that contains signal energy. This characteristic is consistent with the default choice for the sample interval. For a given sample interval length, although in general the interval location is not the optimum one in terms of signal energy, it should be approximately so in most cases.

The program noise parameter is SIG. It represents the standard deviation  $\sigma$  associated with the magnetic noise process of the two encounter models. In terms of the ideal low pass filter implied by the encounter models, its square should be equal to  $\text{MAXF} \cdot (\text{SIGO})^2$  where SIGO is the magnetic noise process constant power spectral density. The program does not enforce this relation. Therefore, in using the program, the implied relation between the two input parameters: magnetic noise and maximum magnetic signal frequency should be kept in mind. If an average value of the peak-to-peak magnetic noise for an encounter can be estimated, for example from a magnetometer trace, then the value for SIG should be chosen so that the estimate is 4 to 6 times this value.

## Appendix 1. The Detection Statistics

In this appendix,  $y_1, y_2, \dots, y_m$  are sequential values (voltages) representing the sample values in a sample interval. They are the input to a magnetometer's detector. With these  $m$  sample values, the detector computes the value of a detection statistic. This value is represented by  $x$  and the detection decision corresponding to the sample interval is determined by the decision rule: If  $x \geq x^*$ , then the input during the sample interval was noise plus signal, otherwise, the input was noise.

For both encounter models, the detection probability and the false alarm probability are decreasing functions of  $x^*$  and the relation is one-to-one in both cases. In the program, the false alarm probability  $p_f$  is used to determine a unique value of the threshold  $x^*$ . This value is then used to determine a unique value of the detection probability  $p_d$ .

In the program,  $p_f$  is found using the relation  $p_f = R \cdot \delta t$  where  $R$  is the false alarm rate in false alarms per second and  $\delta t$  is the sample interval length in seconds. This relation is based on the following argument: With no signal energy in a sample interval,  $y_1 = n_1, y_2 = n_2, \dots, y_m = n_m$  where  $n_1, n_2, \dots, n_m$  are noise values (voltages) input to a magnetometer's detector. The  $n_i$  are values of independent normal (gaussian) random variables, each with mean zero and standard deviation  $\sigma$ . Because of this, in the encounter models, values of  $x$  for different sample intervals are the values of independent random variables that determine two outcomes:  $x \geq x^*$  or  $x < x^*$ .

Therefore, in terms of these outcomes, an encounter is a sequence of independent Bernoulli trials. If there is no signal present, since the noise is determined by a stationary process,  $p_f$  is the same for each sample interval and, in terms of these outcomes, the sequence is a series of repeated independent Bernoulli trials and therefore the expected number of trials between false alarms is  $1/p_f$ . Since the time between trials is  $\delta t$ , the expected number of seconds between false alarm is  $\delta t/p_f$  or the expected number of false alarms per second  $R$  is equal to  $p_f/\delta t$ .

The determination of  $x^*$  depends on the encounter model statistic. For both encounter models, when there is a signal,  $Y_1 = n_1 + s_1$ ,  $Y_2 = n_2 + s_2$ ,  $\dots$ ,  $Y_m = n_m + s_m$  where  $s_1, s_2, \dots, s_m$  are signal values (voltages) input to a magnetometer's detector. The models imply that the signal values  $s_i = K \cdot (H_S)_i$  where  $K$  is a constant whose value is determined by the characteristics of the encounter magnetometer and where the  $(H_S)_i$  are dipole magnetic signal intensities. The models also imply that the noise values  $n_i$  are determined by a gaussian stochastic processes characterized by a standard deviation  $\sigma$  and that  $n_i = K \cdot (H_N)_i + n'_i$  where the  $(H_N)_i$  are magnetic noise intensities and the  $n'_i$  are magnetometer instrument noise values (voltages). In the program, the magnitude of  $K$  is 1. Since, for both models,  $p_d$  depends only on the ratio of signal energy to noise energy for a sample interval, this is a satisfactory choice for the program if instrument noise is assumed to be determined by a process independent of the magnetic noise process



and to be expressed in terms of an equivalent magnetic noise

$$(H)_i^! = (1/K) \cdot n_i^!.$$

For both encounter models, the signal (the average signal power)  $S = (1/m) \cdot \sum s_i^2$  and the noise (the expected value of the average noise power)  $N = \sigma^2$  so that the signal-to-noise ratio is  $(1/m) \cdot \sum s_i^2 / \sigma^2$  where the sum index  $i = 1, 2, \dots, m$ .

**The Crosscorrelation Detector Statistic:** The statistic for the first encounter model is a crosscorrelation detector statistic that is defined by the sum

$$x = \sum y_i \cdot s_i$$

where the summation index  $i = 1, 2, \dots, m$  and the sum is over the values corresponding to a sample interval. For the first encounter model, the characteristics of both the noise and the signal are required in order to determine encounter detection probabilities. In particular, the signal values for an encounter are in the memory of the detector prior to the encounter. For the encounter conditions and a specified false alarm probability, the statistic is optimum in the sense that the encounter detection probability for this statistic is at least equal to that for any other statistic. Because of these considerations, encounter probabilities based on the crosscorrelation statistic can be considered to represent upper bounds on detection performance against dipole targets for magnetometers of the type described by the models.

For a sample interval without signal energy,  $x$  is the value of a normal random variable with a mean  $\mu_x = 0$  and a variance

$\sigma_X^2 = \sigma^2 \cdot \sum s_i^2$  where  $\sigma$  is the standard deviation associated with the noise process and the sum index  $i = 1, 2, \dots, m$  and the sum is over the values corresponding to the sample interval.

This implies that

$$p_f = 1 - P(x^*/\sigma_X)$$

where  $P(z)$  is the standard normal cumulative distribution function. This relation is the basis for determining the threshold value  $x^*$ .

For the sample interval with signal energy,  $x$  is the value of a normal random variable with a mean  $\mu_X = \sum s_i^2$  where the sum index  $i = 1, 2, \dots, m$  and the sum is over the values corresponding to the sample interval. This implies that

$$p_d = 1 - P(v^* - d^{\frac{1}{2}})$$

where  $v^* = x^*/\sigma_X$  and  $d = \sum s_i^2/\sigma^2 = (1/m) \cdot S/N$ . This relation is the basis for determining encounter detection probabilities for the first encounter model. The relation implies that for a specified false alarm probability  $p_f$  the detection probability  $p_d$  is an increasing function of the signal to-noise ratio  $S/N$ .

**The Square Law Detector Statistic:** The statistic for the second encounter model is a square law (energy) detector statistic that is defined by the sum

$$x = \sum y_i^2$$

where the sum index  $i = 1, 2, \dots, m$  and the sum is over the values corresponding to the sample interval. For the second encounter model, only the characteristics of the noise are required to determine encounter detection probabilities.

For a sample interval without signal energy,  $x/\sigma^2$  is the value of a chi-square random variable with  $m$  degrees of freedom. This implies that

$$p_f = 1 - P(x^*/\sigma^2 | m)$$

where  $P(x^*/\sigma^2 | m)$  is the chi-square cumulative distribution function for a chi-square random variable with  $m$  degrees of freedom and where  $\sigma$  is the standard deviation associated with the noise process. This relation is the basis for determining the threshold value  $x^*$ .

For the sample interval with signal energy,  $x/\sigma^2$  is the value of a noncentral chi-square random variable with  $m$  degrees of freedom and noncentral parameter  $\sum s_i^2/\sigma^2$  where the sum index  $i = 1, 2, \dots, m$  and the sum is over the values corresponding to the sample interval. This implies that

$$p_d = 1 - P(x^*/\sigma^2 | m, \sum s_i^2/\sigma^2)$$

where  $P(x^*/\sigma^2 | m, \sum s_i^2/\sigma^2)$  is the noncentral chi-square cumulative distribution function for a noncentral chi-square random variable with  $m$  degrees of freedom and noncentral parameter  $\sum s_i^2/\sigma^2 = (1/m) \cdot S/N$ . This relation is the basis for determining encounter detection probabilities for the second encounter model. The relation implies that for a specified false alarm probability  $p_f$ , the detection probability  $p_d$  is an increasing function of the signal-to-noise ratio  $S/N$ . This is made more evident by the following relation:

$$P(x^*/\sigma^2 | m, \sum s_i^2/\sigma^2) = \sum \{ (a^j/j!) \cdot \exp(-a) \cdot P[x^*/\sigma^2 | (m + 2 \cdot j)] \}$$

where  $a = (1/2) \cdot \sum s_i^2/\sigma^2 = (m/2) \cdot (S/N)$ , the sum index  $j = 1, 2,$



$\dots$ ,  $m$  and the sum index  $j = 0, 1, 2, \dots$ . (Note,  $p_d \geq p_f$ ,  
as expected, since  $P(x^*/\sigma^2|m) \leq P[x^*/\sigma^2|(m + 2 \cdot j)]$  for  $j = 0,$   
 $1, 2, \dots$ .)

## Appendix 2. Program Probability Calculations

The program evaluates the cumulative and inverse cumulative distribution functions using approximations described in Reference 2. These approximations are listed below.

### The Standard Normal Cumulative Distribution Function

#### Approximation:

$$P(z) = 1 - s \cdot t \cdot (b_1 + t \cdot (b_2 + t \cdot (b_3 + t \cdot (b_4 + t \cdot b_5))))$$

where  $s = (1/(2 \cdot \pi)^{\frac{1}{2}}) \cdot \exp(-z^2/2)$  and  $t = 1/(1 + b_0 \cdot z)$ . And where

$$b_0 = .2316419, \quad b_1 = .319381530, \quad b_2 = -.356563782,$$

$$b_4 = -1.821255978, \quad b_5 = 1.330274429,$$

and  $z \geq 0$ . For  $z < 0$ ,  $P(z) = 1 - P(|z|)$ .

### The Inverse Standard Normal Cumulative Distribution Function

#### Approximation:

$$z(P) = t - (c_0 + t \cdot (c_1 + t \cdot c_2)) / (1 + t \cdot (d_1 + t \cdot (d_2 + t \cdot d_3)))$$

where  $t = (\ln(1/Q^2))^{\frac{1}{2}}$  and  $Q = 1 - P$ . And where

$$c_0 = 2.515517, \quad c_1 = .802853, \quad c_2 = .010328,$$

$$d_1 = 1.432788, \quad d_2 = .189269, \quad d_3 = .001308,$$

and  $.5 \leq P < 1$ . For  $0 < P < .5$ ,  $z(P) = -z(1 - P)$ .

### The Inverse Chi-Square Cumulative Distribution Function

#### Approximation:

$$v(P|m) = m \cdot [1 - 2/(9 \cdot m) + z \cdot (2/(9 \cdot m))^{\frac{1}{2}}]^3$$

where  $P(z) = P(v|m)$ . In the program, the inverse standard normal cumulative distribution function approximation is used to determine  $z$ .

## The Noncentral Chi-Square Cumulative Distribution Function

Approximation:

$$P(w|m, \sum s_i^2/\sigma^2) = P(z)$$

where  $z = [2 \cdot w / (1 + b)]^{\frac{1}{2}} - [2 \cdot a / (1 + b) - 1]^{\frac{1}{2}}$  with  
 $a = m + \sum s_i^2/\sigma^2$ ,  $b = (\sum s_i^2/\sigma^2) / (m + \sum s_i^2/\sigma^2)$  and the sum index  
 $i = 1, 2, \dots, m$ . In the program, the standard normal  
cumulative distribution function approximation is used to  
determine  $P(z)$ .

### Appendix 3. The Magnetic Signal

The encounter models are defined by the following conditions: A submarine magnetic anomaly field is a magnetic dipole field that is superimposed on an earth magnetic field that is constant over an encounter region. A magnetometer magnetic signal value is the magnitude of the projection of a dipole magnetic field on the earth magnetic field at the location of the magnetometer. The basis for determining magnetic signal values in the program is an expression that involves the magnetic dipole moment, the earth magnetic field and the position of the magnetometer relative to the dipole.

The expression can be developed as follows: In the rectangular coordinate system that is shown in Figure 1, a magnetic dipole is at the origin, the xy-plane is the horizontal plane at a representative point in an encounter region, the positive y-axis is in the direction of magnetic north, the positive x-axis is in the direction of magnetic east and the positive z-axis is positive upward. In this rectangular coordinate system, the constant earth magnetic field can be expressed by  $H_e = H_e \cdot (j \cdot \cos \Phi + k \cdot \sin \Phi)$  where  $\Phi$  is a magnetic dip angle and  $H_e$  is a magnetic field magnitude that characterizes the earth field in an encounter region. In a spherical coordinate system with the origin at the magnetic dipole and the polar axis in the direction of the dipole moment,  $H_d = (c \cdot p / r^3) \cdot (2 \cdot r \cdot \cos \theta + \theta \cdot \sin \theta)$  is the magnetic field of the dipole at a point with spherical coordinates  $(r, \theta, \Gamma)$ .

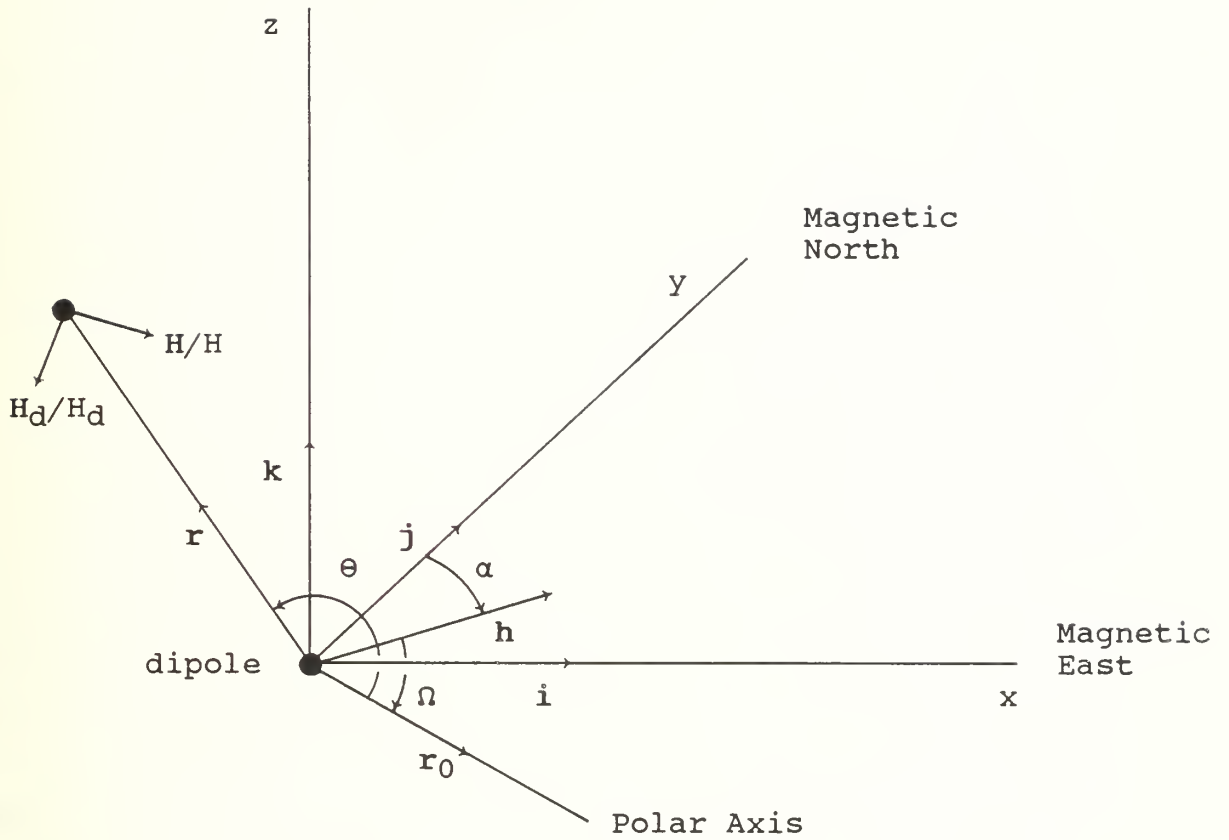


Figure 1. A unit vector in the direction of the earth field and a unit vector in the direction of the dipole field of a dipole at the origin are shown at a point that is a distance  $r$  from the dipole. The unit vector  $h$  is in the direction of the horizontal component of the dipole moment.

In this expression  $p$  is the magnitude of the dipole moment and  $c$  is a constant whose value is determined by the choice of units. The magnetic signal for a magnetometer that is described by the encounter models is  $H_S = H_e \cdot H_d / H_e$  when the magnetometer is at the point  $(r, \theta, \Gamma)$ .  $H_S$  can be expressed in terms of the rectangular coordinate system as follows: First, let  $r_0$  be a unit vector in the direction of the spherical coordinate system polar axis. Then the magnetic dipole field

$$H_d = (c \cdot p / r^3) \cdot (3 \cdot r \cdot \cos \theta - r_0)$$

since  $r = (i \cdot x + j \cdot y + k \cdot z) / r$  where  $r = (x^2 + y^2 + z^2)^{1/2}$ ,  $\theta = [ (r_0 \times r) \times r ] / \sin \theta = [ (r \cdot r_0) \cdot r - (r \cdot r) \cdot r_0 ] / \sin \theta$ , and the dot product  $r \cdot r = \cos \theta$ . The unit vector  $r_0$  can be expressed in the rectangular coordinates by noting that  $r_0 = p / p$  and then expressing  $p$  in rectangular coordinates. To do this, let  $\Omega$  be the depression angle of  $p$  from the xy-plane (the horizontal plane) with  $\Omega$  positive downward and let  $\alpha$  be the direction of  $p$  relative to magnetic north. Then the magnetic dipole moment  $p = p \cdot (h \cdot \cos \Omega - k \cdot \sin \Omega)$  in terms of the unit vector  $k$  and the unit vector  $h = i \cdot \sin \alpha + j \cdot \cos \alpha$  which has the direction of the horizontal component of  $p$ . With these results, the unit vector  $r_0 = i \cdot (\cos \Omega \cdot \sin \alpha) + j \cdot (\cos \Omega \cdot \cos \alpha) - k \cdot \sin \Omega$  and

$$H_S = (c \cdot p / r^3) \cdot [ (3/r) \cdot (\cos \theta) \cdot (y \cdot \cos \phi - z \cdot \sin \phi) - (\cos \phi \cdot \cos \Omega \cdot \cos \alpha + \sin \phi \cdot \sin \Omega) ]$$

where  $\cos \theta = (1/r) \cdot (x \cdot \cos \Omega \cdot \sin \alpha + y \cdot \cos \Omega \cdot \cos \alpha - z \cdot \sin \Omega)$  since  $\cos \theta = r \cdot r_0$ . As can be seen from this expression, for a constant dipole moment magnitude and direction and a constant

earth field magnitude and direction, the magnetic signal is only a function of the rectangular coordinates of the location of the magnetometer relative to the dipole. In the encounter models, both of these conditions are satisfied. However, by allowing  $p$ ,  $\Omega$ ,  $\alpha$ ,  $H_S$  and  $\Phi$  to vary, the expression for  $H_S$  is applicable to more general encounter models.

#### Appendix 4. The Anderson Formulation

In the encounter models, the magnetic signal  $H_S$  at a sample point in a straight line encounter can be represented in a form described by Anderson in Reference 4. For convenience, the Anderson formulation is used in the program to determine values for  $H_S$ . It can be developed as follows: The primed rectangular coordinate system that is shown in Figure 2 is superimposed on the rectangular coordinate system of Figure 1 so that the origin is coincident with the origin of that system. A magnetometer is in a straight line encounter with a magnetic dipole that is located at the origin of the combined system. The combined system moves with the magnetic dipole with the  $x'$ -axis oriented so that it is parallel to and in the direction of the track of the magnetometer relative to the magnetic dipole and the  $z'$ -axis oriented so that it is directed toward and passes through the CPA on that track. Let  $l$ ,  $m$ , and  $n$  be the direction cosines of the dipole moment  $\mathbf{p}$  and  $l_1$ ,  $m_1$  and  $n_1$  be the direction cosines of the earth magnetic field  $H_E$ . Then, the unit vector  $\mathbf{r}_0 = \mathbf{i}' \cdot l + \mathbf{j}' \cdot n + \mathbf{k}' \cdot m$ . And, for points on the relative track,  $x' = s'$ ,  $y' = 0$  and  $z' = R$  where  $R$  is the slant range of the dipole at CPA and where  $s'$  is the algebraic distance of the magnetometer from CPA on the relative track. (It is negative for points before CPA and positive for points after CPA.) This implies that magnetometer position vector in the



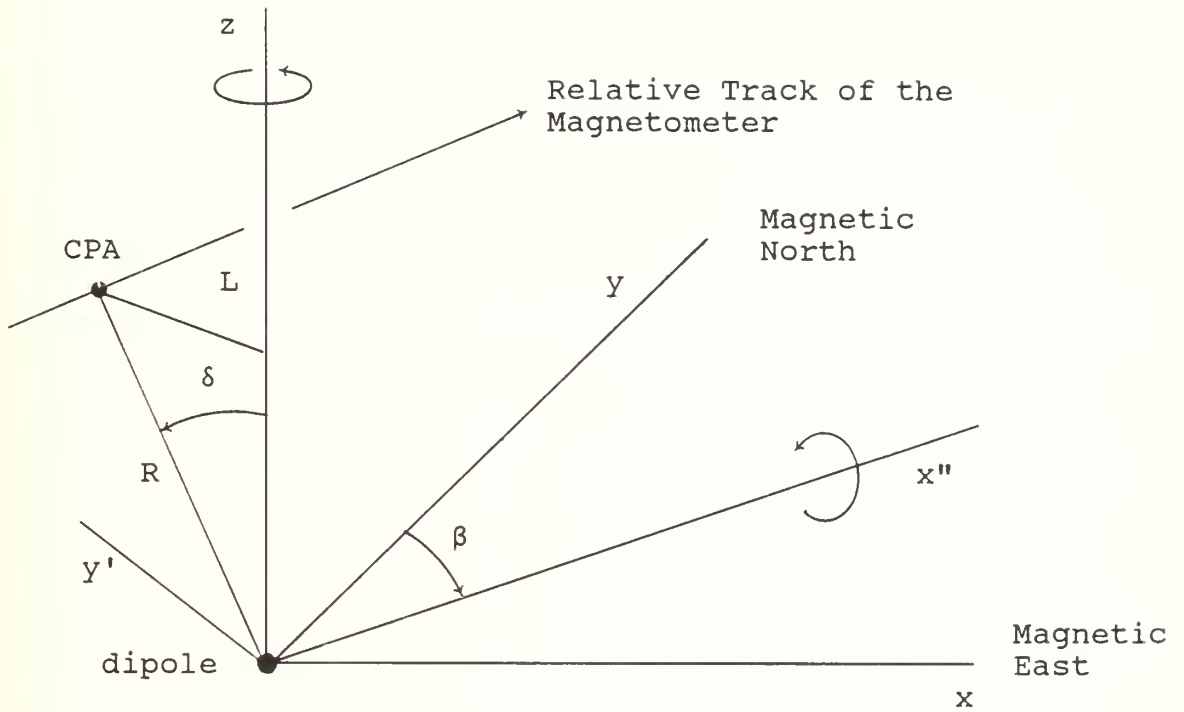


Figure 2. The dipole is at the origin in the unprimed, primed and double primed coordinate systems. In the primed system, the coordinates of the magnetometer are  $(s', 0, R)$  and the coordinates of the CPA are  $(0, 0, R)$ .

moving coordinate system is  $\mathbf{r} = \mathbf{i}' \cdot (s'/r) + \mathbf{k}' \cdot (R/r)$ . From Appendix 3,

$$H_d = (c \cdot p / r^3) \cdot (3 \cdot \mathbf{r} \cdot \cos \theta - r_0).$$

With  $\mathbf{r}$  and  $r_0$  expressed in terms of the primed unit vectors and  $\cos \theta = \mathbf{r} \cdot \mathbf{r}_0 = l \cdot (s'/r) + n \cdot (R/r)$ , this becomes

$$H_d = (c \cdot p / r^3) \cdot [(3/r^2) \cdot (l \cdot s' + n \cdot R) \cdot (\mathbf{i}' \cdot s' + \mathbf{k}' \cdot R) - (\mathbf{i}' \cdot l + \mathbf{j}' \cdot m + \mathbf{k}' \cdot n)].$$

Then, since  $H_e/H_e = \mathbf{i}' \cdot l_1 + \mathbf{j}' \cdot m_1 + \mathbf{k}' \cdot n_1$  and  $H_s = H_e \cdot H_d / H_e$ ,

$$H_s = (c \cdot p / r^3) \cdot [(2 \cdot l \cdot l_1 - m \cdot m_1 - n \cdot n_1) \cdot (s')^2 + 3 \cdot (n \cdot l_1 + l \cdot n_1) \cdot s' \cdot R + (2 \cdot n \cdot n_1 - l \cdot l_1 - m \cdot m_1) \cdot R^2].$$

The quantities

$$A_2 = 2 \cdot l \cdot l_1 - m \cdot m_1 - n \cdot n_1$$

$$A_1 = 3 \cdot (n \cdot l_1 + l \cdot n_1)$$

$$A_0 = 2 \cdot n \cdot n_1 - l \cdot l_1 - m \cdot m_1,$$

are called the Anderson coefficients. With  $r = [(s')^2 + R^2]^{1/2}$  and  $\beta = s'/R$ ,  $H_s$  can now be expressed as follows:

$$H_s(\beta) = (c \cdot p / R^3) \cdot \sum A_j \cdot F_j(\beta)$$

where the  $F_j(\beta) = \beta^j / (1 + \beta^2)^{5/2}$  for  $j = 0, 1, 2$  are called the Anderson functions. This is the Anderson formulation.

To relate the Anderson formulation for  $H_s$  to the formulation for  $H_s$  in Appendix 3, first note that

$$l = r_0 \cdot \mathbf{i}'$$

$$m = r_0 \cdot \mathbf{j}'$$

$$n = r_0 \cdot \mathbf{k}'$$

and

$$l_1 = (H_e/H_e) \cdot i'$$

$$m_1 = (H_e/H_e) \cdot j'$$

$$n_1 = (H_e/H_e) \cdot k'.$$

Then express  $r_0$ ,  $(H_e/H_e)$  and the unit vectors  $i'$ ,  $j'$  and  $k'$  in terms of the unit vectors  $i$ ,  $j$  and  $k$  and take the indicated dot products. From Appendix 3,

$$r_0 = i \cdot (\cos \Omega \cdot \sin \alpha) + j \cdot (\cos \Omega \cdot \cos \alpha) - k \cdot \sin \Omega$$

and

$$H_e/H_e = j \cdot \cos \Phi + k \cdot \sin \Phi.$$

To express  $i'$ ,  $j'$  and  $k'$  in terms of the unit vectors  $i$ ,  $j$  and  $k$ , note that the unprimed coordinate system can be transformed to the primed coordinate system by two rotations that are defined as follows: First, rotate a coordinate system that is coincident with the unprimed coordinate system about the z-axis through the angle  $(\phi - \pi/2)$  with positive angles clockwise (left hand rule) so that its x-axis is parallel to and in the direction of the relative track. Next, rotate this system about its x-axis through an angle  $\delta$  with positive angles counterclockwise (right hand rule) so that the positive z-axis passes through the CPA. The angle  $\delta$  is related to the vertical separation  $z_0$  of the magnetometer and the dipole and the algebraic encounter lateral range  $L$  that is positive if the dipole is to the left of the relative track. With these sign definitions:  $L = R \cdot \sin \delta$  and  $z_0 = R \cdot \cos \delta$ . After the rotation, the auxiliary coordinate system is coincident with the primed coordinate system.

These transformations can be described in terms of matrix equations as follows: Let  $(x'', y'', z'')$  be the coordinates of a point in the coordinate system that is coincident with the auxiliary coordinate system after the first rotation. Then the transformation from the unprimed coordinate system to this double primed coordinate system is described by the matrix equation

$$\begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix} = \begin{vmatrix} \sin \phi & \cos \phi & 0 \\ -\cos \phi & \sin \phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}.$$

And the transformation from the double primed coordinate system to the primed coordinate system is described by the matrix equation

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \delta & \sin \delta \\ 0 & -\sin \delta & \cos \delta \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix}.$$

Taking the product of the rotation matrices in the indicated order yields the matrix equation

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} \sin \phi & \cos \phi & 0 \\ -\cos \delta \cos \phi & \cos \delta \sin \phi & \sin \delta \\ \sin \delta \cos \phi & -\sin \delta \sin \phi & \cos \delta \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

which defines the transformation from the unprimed to the primed coordinate system. The unprimed vector components of the unit vector  $i'$  can be found by transforming the coordinates  $(1, 0, 0)$

in the primed system to their corresponding coordinates in the unprimed system with the inverse of this matrix and then repeating this process for  $(0,1,0)$  and  $(0,0,1)$  in order to find the unprimed unit vector components of  $j'$  and  $k'$ . However, since the inverse transformation matrix is the transpose of this matrix, the elements of the row that corresponds to a primed unit vector are the magnitudes of the unprimed vectors that are its components. Consequently:

$$i' = i \cdot \sin \phi + j \cdot \cos \phi$$

$$j' = -i \cdot \cos \delta \cos \phi + j \cdot \cos \delta \sin \phi + k \cdot \sin \delta$$

$$k' = i \cdot \sin \delta \cos \phi - j \cdot \sin \delta \sin \phi + k \cdot \cos \delta.$$

Then, taking the dot products between  $r$ ,  $r_0$  and these three unit vectors as indicate above gives:

$$l = \cos \Omega \cdot \cos (\phi - \alpha)$$

$$m = \cos \delta \cdot \cos \Omega \cdot \sin (\phi - \alpha) - \sin \delta \cdot \sin \Omega$$

$$n = -\sin \delta \cdot \cos \Omega \cdot \sin (\phi - \alpha) - \cos \delta \cdot \sin \Omega$$

and

$$l_1 = \cos \Phi \cdot \cos \phi$$

$$m_1 = \cos \delta \cdot \cos \Phi \cdot \sin \phi - \sin \delta \cdot \sin \Phi$$

$$n_1 = -\sin \delta \cdot \cos \Phi \cdot \sin \phi - \cos \delta \cdot \sin \Phi.$$

These are the relations that are used in the program to determine values for the Anderson coefficients.

## Appendix 5. The Encounter Equations of Motion

In the double primed coordinate system that is defined in Appendix 4, the equations of motion of a magnetometer relative to a submarine (dipole) are:

$$x''(t) = s'(t)$$

$$y''(t) = -L$$

$$z''(t) = z_0.$$

where  $L$  is the algebraic encounter lateral range that is defined in Appendix 4,  $z_0$  is the vertical separation between the magnetometer and the submarine and  $s'(t)$  is the distance of the magnetometer from the CPA on the relative track. With  $w$  the speed of the magnetometer relative to the submarine and  $t$  a relative time parameter,  $s'(t) = w \cdot t$ . These equations can be considered to be the ones used in the program to describe the motion of a magnetometer relative to a submarine. There,  $t$  is determined by  $t = [j - (m-1)/2] \cdot \delta t$  where the index  $j = 1, 2, \dots, m$  and  $\delta t$ , a time step, is the time between samples. Note, when  $t = 0$ , the magnetometer is at the CPA.

In the coordinate system of Figure 1 in a straight line encounter as defined in the encounter models, the equations of motion of a magnetometer relative to a submarine can be written as follows:

$$x(t) = s'(t) \cdot \sin \phi + L \cdot \cos \phi$$

$$y(t) = s'(t) \cdot \cos \phi - L \cdot \sin \phi$$

$$z(t) = z_0,$$

since the transformation from the double primed coordinate system to the primed coordinate system is determined by the matrix equation

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} \sin \phi & -\cos \phi & 0 \\ \cos \phi & \sin \phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix},$$

$x'' = s'(t)$  and  $y'' = -L$ . The above equations and the expression for  $H_S$  in Appendix 3 could have been used in the program to evaluate the magnetic signal. In particular, with these equations of motion and with the two relations  $L = R \cdot \sin \delta$  and  $z_0 = R \cdot \cos \delta$ , the expression for  $H_S$  in Appendix 3 can be written in terms of  $\Omega, \alpha, \Phi, \phi, \delta$  and  $R$  so that it is identical in appearance to the Anderson formulation for  $H_S$  in terms of these quantities. The definition of  $\delta$  in Appendix 4 in terms of a counterclockwise rotation results in a definition of the algebraic lateral range that is consistent with some that have been used elsewhere.

In the program, the relation  $w = v - u$  is the basis for determining the relative speed  $w$ . In this relation,  $v$  is the velocity of the magnetometer,  $u$  is the velocity of the submarine (dipole) and  $w$  is the velocity of the magnetometer relative to the submarine. This relation implies the following equations:  $w_x = v_x - u_x$ ,  $w_y = v_y - u_y$  and  $w_z = v_z - u_z$  where the coordinates  $x, y$  and  $z$  refer to a fixed coordinate system with the same orientation as that of Figure 1. In the encounters



of the models,  $v_x = v \cdot \sin \sigma$ ,  $v_y = v \cdot \cos \sigma$  and  $v_z = 0$  where  $\sigma$  is the magnetic course and  $v$  is speed of the magnetometer. And,  $u_x = u \cdot \sin \beta$ ,  $u_y = u \cdot \cos \beta$  and  $u_z = 0$  where  $\beta$  is the magnetic course and  $u$  is the speed of the submarine. The relative magnetic course  $\phi$  and the relative speed of the magnetometer are defined by  $w_x = w \cdot \sin \phi$  and  $w_y = w \cdot \cos \phi$ . In the program,  $\phi$  and  $w$  are determined with a rectangular to polar conversion routine where  $w = (w_x^2 + w_y^2)^{\frac{1}{2}}$  and where  $\phi$  is determined by  $\sin^{-1}(w_x/w)$  and  $\cos^{-1}(w_y/w)$ .



## Appendix 6. The Submarine Magnetic Dipole

In the encounter models, the magnitude and direction of a submarine's dipole moment  $\mathbf{p}$  are determined by first determining its components in the rectangular coordinate system of Figure 1. In that coordinate system,

$$p_x = (p_{LP} + p_{LI}) \cdot \sin \beta + (p_{TP} + p_{TI}) \cdot \cos \beta$$

$$p_y = (p_{LP} + p_{LI}) \cdot \cos \beta - (p_{TP} + p_{TI}) \cdot \sin \beta$$

$$p_z = -(p_{VP} + p_{VI})$$

where  $\beta$  is the submarine's magnetic course,  $p_{LP}$ ,  $p_{TP}$  and  $p_{VP}$  are the permanent and  $p_{LI}$ ,  $p_{TI}$  and  $p_{VI}$  are the induced longitudinal, transverse and vertical magnetic dipole moments of the submarine. These relations are based on the following sign convention:

$p_L$  is positive when  $\mathbf{p}_L$  is directed from stern to bow.

$p_T$  is positive when  $\mathbf{p}_T$  is directed from port to starboard.

$p_V$  is positive when  $\mathbf{p}_V$  is directed downward.

The permanent dipole moments are input parameters in the program and the induced dipole moments are determined in a way that is similar to one that is described in Reference 1. In the encounter models, a submarine is a ferromagnetic prolate ellipsoid with the major axis the submarine's longitudinal axis and the equal minor axes the submarine's transverse and vertical axes. And, the induced dipole moments are:

$$p_{LI} = k_L \cdot H_{eL}$$

$$p_{TI} = k_T \cdot H_{eT}$$

$$p_{VI} = k_V \cdot H_{eV}$$

where  $H_{eL}$ ,  $H_{eT}$  and  $H_{eV}$  are the vector components of the earth magnetic field in the rectangular coordinate system defined by the ellipsoid axes and the submarine magnetic moment sign convention. The magnitude of these vector components are:

$$H_{eL} = H_e \cdot \cos \Phi \cos \beta$$

$$H_{eT} = -H_e \cdot \cos \Phi \sin \beta$$

$$H_{eV} = H_e \cdot \sin \Phi.$$

The earth magnetic field dip angle  $\Phi$  and the earth magnetic field magnitude  $H_e$  can each be chosen to be input parameters in the program or they can be computed by the program as described in Appendix 7. (Values for  $\Phi$  and  $H_e$  can be found from magnetic data charts, for example, see References 5 and References 6.)

By using the above relations,

$$p_x = H_e \cdot \cos \Phi \cdot (k_L - k_T) \cdot \sin \beta \cos \beta + (p_{LP} \cdot \sin \beta + p_{TP} \cdot \cos \beta)$$

$$p_y = H_e \cdot \cos \Phi \cdot (k_L \cdot \cos^2 \beta + k_T \cdot \sin^2 \beta) + (p_{LP} \cdot \cos \beta - p_{TP} \cdot \sin \beta)$$

$$p_z = -(H_e \cdot k_V \cdot \sin \Phi + p_{VP})$$

In the encounter models, the values of the permeability coefficients  $k_L$ ,  $k_T$  and  $k_V$  are related to submarine displacement by the following relations:

$$k_L = f_L \cdot W$$

$$k_T = f_T \cdot W$$

$$k_V = f_V \cdot W$$

where  $W$  is the submarine displacement in tons and  $f_L$ ,  $f_T$  and  $f_V$  are permeability factors that are determined by a submarine's magnetic characteristics. In the program, the units of  $H_e$  are gamma, the units of  $p$  are in oersted-centimeter<sup>3</sup>, the units of

$k$  are oersted-centimeter<sup>3</sup>/gamma and the units of  $f$  are oersted-centimeter<sup>3</sup>/gamma-ton. If the units of  $H_e$  were gamma, but the units of  $p$  were gamma-foot<sup>3</sup>, then the units of  $k$  would be foot<sup>3</sup> and the units of  $f$  would be foot<sup>3</sup>/ton. To convert  $p$  in gamma-foot<sup>3</sup> to oersted-centimeter<sup>3</sup> or  $k$  in foot<sup>3</sup> to oersted-centimeter<sup>3</sup>/gamma or  $f$  in foot<sup>3</sup>/ton to oersted-centimeter<sup>3</sup>/gamma-ton, divide by 3.53. (The program default values for  $f_L$ ,  $f_T$  and  $f_V$  are values from Reference 1 in foot<sup>3</sup>/ton that have been divided by 3.53 to give values in oersted-centimeter<sup>3</sup>/gamma-ton.) To convert  $p$  in weber-meter to oersted-centimeter<sup>3</sup>, multiply by  $[1/(4 \cdot \pi)] \cdot 10^{10}$  and to convert  $p$  in ampere-centimeter<sup>2</sup> to oersted-centimeter<sup>3</sup>, multiply by  $1 \cdot 10^3$ .

## Appendix 7. The Earth Magnetic Field and Dip Angle

An auxiliary magnetic field model is described in this appendix. The model is the basis for a default choice for either the value of the earth magnetic field magnitude parameter  $H_e$  or the earth magnetic field dip angle parameter  $\Phi$ . Relative to encounter model accuracy, the default values should be adequate in most cases.

In the model, the earth magnetic field is generated by a magnetic dipole that is located at the earth's center and the earth is a nonmagnetic sphere of radius  $r_e$ . With  $p_e$  the magnitude of the dipole moment, the magnitude of the earth field at a point is

$$H_e = (H_e \cdot H_e)^{\frac{1}{2}} = (c \cdot p_e / r^3) \cdot (3 \cdot \cos^2 \theta + 1)^{\frac{1}{2}}$$

where  $\theta$  is the polar angle of the point in a spherical coordinate system and  $c$  is a constant whose value is determined by the choice of units. The dipole moment is coincident with and in the direction of the polar axis which is directed toward the earth's southern hemisphere. In this coordinate system, at any point on the surface of the earth:

$$H_e = H_{e0} \cdot (3 \cdot \cos^2 \theta + 1)^{\frac{1}{2}}$$

where  $H_{e0}$  is the value of  $H_e$  at the magnetic equator which is defined by the points on the earth where  $\theta = 90^\circ$ . In terms of the dip angle, at any point on the earth's surface:

$$H_e = 2 \cdot H_{e0} \cdot (3 \cdot \cos^2 \Phi + 1)^{-\frac{1}{2}}.$$

This expression can be obtained by noting that  $\Phi$  can be

defined in terms of the or the  $r$  and  $\theta$  components of  $H_e$  in the spherical coordinates as follows:

$$\sin \Phi = -H_{er}/H_e = -2 \cdot H_{e0} \cdot (\cos \theta)/H_e$$

and

$$\cos \Phi = H_{e\theta}/H_e = H_{e0} \cdot (\sin \theta)/H_e.$$

Based on these relations,

$$[(\sin \Phi)/2]^2 + \cos^2 \Phi = (H_{e0})^2$$

and

$$H_e = H_{e0} \cdot [(\sin^2)/4 + \cos^2 \Phi]^{-\frac{1}{2}} = 2 \cdot H_{e0} \cdot (3 \cdot \cos^2 \Phi + 1)^{-\frac{1}{2}}$$

The dip angle is determined from a magnetic latitude for the encounter region.

The magnetic latitude and longitude of the representative point of an encounter region can be defined in terms of its geographic latitude and longitude by using the following transformations: First, convert the latitude and longitude of the point to rectangular coordinates in a right-handed coordinate system whose origin is at the center of a spherical earth, whose positive z-axis passes through its north geographic pole and whose positive y-axis passes through the point on its equator with latitude  $0^\circ$  and longitude  $0^\circ$ . Next, rotate a coordinate system that is coincident with this system in a clockwise direction (left hand rule) about its z-axis so that its positive y-axis passes through the point with latitude  $0^\circ$  and with longitude equal to that of the north magnetic pole. Then, rotate the system in a clockwise direction about its x-axis so that its positive z-axis passes through the north magnetic pole. Next,

transform the rectangular coordinates of the representative point in this system to its coordinates in the spherical coordinate system that is associated with it. Then, with  $\Omega$  the polar angle of the representative point in this system, the magnetic latitude of the representative point is  $L_M = 90^\circ - \Omega$ . Since  $\tan \Phi = -2 \cdot \cot \Theta$  from above and  $\Theta = 180^\circ - \Omega$ , the dip angle  $\Phi$  is given by the following relation:

$$\Phi = \tan^{-1}(2 \cdot \tan L_M).$$

In the program, the transformations described above are accomplished in part by a rectangular to polar conversion routine. In particular, by rotating the final rectangular coordinate system about its z-axis so that the x-coordinate is zero, the polar angle of the polar coordinates of the representative point in the resulting yz-plane determines  $L_M$ .

Some values of  $\Phi$  and  $H_e$  that are listed in Table 1 were generated using the program. The latitude and longitude of the magnetic pole that are in the program are  $76^\circ$  N and  $100^\circ$  W. Values of  $\Phi$  from Reference 6 and of  $H_e$  from Reference 7 are also listed in Table 1. A comparison of the program values with these values gives an indication of the errors inherent in the procedure.



Encounter Latitude	Encounter Longitude	Program Dip $\Phi$	Chart #30 Dip $\Phi$	Program Field $H_e$	Chart #39 Field $H_e$
60°N	180°W	74°	70°	.63	.53
60°N	150°W	78°	73°	.66	.55
60°N	90°W	82°	84°	.68	.61
60°N	30°W	75°	74°	.64	.52
30°N	150°W	58°	50°	.51	.41
30°N	90°W	62°	60°	.55	.51
30°N	60°W	59°	59°	.53	.48
30°N	30°E	37°	43°	.41	.42
30°N	60°E	31°	45°	.39	.46
30°N	90°E	30°	44°	.39	.44
30°N	150°E	42°	40°	.43	.40
0°	90°W	26°	18°	.38	.32
0°	30°W	9°	-5°	.35	.28
0°	0°	-48°	-25°	.35	.31
0°	60°E	-25°	-20°	.38	.37
30°S	180°W	-45°	-55°	.44	.50
30°S	90°W	-30°	-34°	.39	.31
30°S	90°E	-62°	-66°	.55	.52
60°S	30°W	-69°	-56°	.60	.34

Table 1. Some corresponding values for the encounter dip angle  $\Phi$  and the earth magnetic field intensity  $H_e$  in gamma. Chart #33 refers to Reference 5 and Chart #39 refers to Reference 6.

A small difference between a chart value and a computed value for the dip angle  $\Phi$  at some location does not imply a small difference between the chart value and the computed value for the earth magnetic field intensity at that location. For example, note the values for 60°N latitude and 30°W longitude. (Positive values of  $\Phi$  indicate the inclination or dip below the horizontal of the north seeking end of a dip needle. Negative values indicate the inclination below the horizontal of the south seeking end.)

An extension of the above procedure can be made for finding the magnetic variation at a location. However, the values generated by using the procedure are generally unsatisfactory. Magnetic variation values are charted in Reference 7.



## Appendix 8. An Alternative Encounter Model

The alternative model that is referred to in Section III is described in more detail in this appendix. In the model, the detection range is the slant range  $R$  at the CPA for an encounter with a specified detection probability (usually .5) is defined by:

$$R = [c \cdot p / H_S]^{1/3}$$

where  $c$  and  $p$  are defined in Appendix 3 and  $H_S$  represents a minimum detectable average magnetic signal that is defined by:

$$H_S = (ORF) \cdot N_M$$

where  $ORF$  is a signal-to-noise ratio called the operator recognition factor and  $N_M$  represents the magnetic noise. Combining these two relations gives:

$$R = \{c \cdot p / [(ORF) \cdot N_M]\}^{1/3}.$$

The value for  $ORF$  depends both on the specified detection probability and on a specified or implied false alarm probability.

The Anderson formulation is consistent with these relations in an approximate sense if the average magnetic signal  $H$  is defined as a root mean square value such that  $H = (c \cdot p / R^3) \cdot k$  and  $k$  is an encounter parameter defined by:

$$k = \{\sum [\sum A_j \cdot F_j(\beta_i)]^2\}^{1/2}$$

where the first sum index  $i = 1, 2, \dots, m$  and the second sum index  $j = 0, 1, 2$ . For a particular encounter geometry,  $k$  is constant and this suggests that the two encounter models could be used to determine an average value for  $k$  for an encounter

region based on average submarine magnetic characteristics. Values for both  $k$  and  $H$  are generated by the program and such values can give an indication of the magnitude of the differences in detection range estimates that are based on this model and either of the other two encounter models. A more detailed comparison of these encounter models is described in Reference 8.

## Appendix 9. An Example of the Program Output

The program is designed to generate the following quantities: encounter parameter values, lateral range function values for the crosscorrelation encounter model and for the square law encounter model, average magnetic signal values, slant range at CPA values, encounter parameter values and magnetic signal values. These values can be saved as a program data file and/or they can be printed.

An example of the program's printed output is listed in Table 2, Table 3 and Table 4. Figure 3 is a plot of the lateral range function values (a lateral range curve) for the square law detector that are listed in Table 3. Figure 4 is a plot of the magnetic signal values that are listed in Table 4.

program file name	MAD.BAS
program data file name	data.mad
magnetic data file name	data.mag
processing data file name	data.pro
kinematic data file name	data.kin
combined magnetic, processing & kinematic data file name	data.mpk
encounter latitude (decimal degrees)	45
encounter longitude (decimal degrees)	-45
encounter variation (decimal degrees)	-25
encounter dip angle (decimal degrees)	68.41608
encounter magnetic field intensity (gamma)	59035.05
target permanent longitudinal moment (oersted-cm3)	0
target permanent transverse moment (oersted-cm3)	0
target permanent vertical moment (oersted-cm3)	0
target displacement (tons)	1800
target longitudinal permeability coefficient	13140
target transverse permeability coefficient	2880
target vertical permeability coefficient	2880
target longitudinal permeability factor	7.3
target transverse permeability factor	1.6
target vertical permeability factor	1.6
sampling period (seconds)	.5
integration time (seconds)	20
adjusted integration time (seconds)	20.5
number of samples per encounter	41
magnetometer course (decimal degrees)	290
magnetometer speed (knots)	180
magnetometer altitude (meters)	100
target course (decimal degrees)	20
target speed (knots)	10
target depth (meters)	100
magnetometer relative magnetic course (decimal degrees)	311.8202
magnetometer relative speed (knots)	180.2776
magnetometer-target vertical separation (meters)	200
target induced longitudinal dipole moment (oersted-cm3)	2.017796E+08
target induced transverse dipole moment (oersted-cm3)	-4.422566E+07
target induced vertical dipole moment (oersted-cm3)	1.58099E+08
target dipole moment (oersted-cm3)	2.601273E+08
target dipole moment azimuth (decimal degrees)	32.63751
target dipole moment depression (decimal degrees)	37.42884
distance between samples on the relative track (meters)	46.37141
false alarm rate (false alarms per hour)	3
false alarm probability	1.666667E-02
magnetic noise (gamma)	.35
maximum lateral range (meters)	500
lateral range step (meters)	20
number of lateral range function values	51

Table 2. An example of a encounter parameter values printout.

L meters	p(cc)	p(sl)	H gamma	R meters	k
-500	.2155959	.0255824	.184247	538.5165	2.818488
-480	.2721803	.029692	.21034	520	2.879916
-460	.3464673	3.582633E-02	.2410144	501.5974	2.943744
-440	.4415205	4.529992E-02	.2785504	483.3218	3.009938
-420	.5573696	6.047795E-02	.3231911	465.1881	3.078418
-400	.6872897	.0856887	.3764825	447.2136	3.149025
-380	.8146549	.1287804	.4404623	429.4182	3.221497
-360	.9159235	.2030898	.5177304	411.8252	3.29544
-340	.9745625	.3272475	.6108596	394.4617	3.370274
-320	.9958122	.5141411	.7233319	377.3593	3.445185
-300	.9997211	.7388175	.8597726	360.5551	3.519032
-280	.9999951	.9177597	1.025158	344.093	3.590261
-260	1	.9896971	1.224529	328.0244	3.656775
-240	1	.999714	1.463644	312.41	3.71578
-220	1	.9999992	1.74981	297.3214	3.763606
-200	1	1	2.085394	282.8427	3.795517
-180	1	1	2.470496	269.0725	3.805514
-160	1	1	2.901352	256.125	3.786234
-140	1	1	3.359555	244.1311	3.72898
-120	1	1	3.819509	233.2381	3.624058
-100	1	1	4.239317	223.6068	3.461596
-80	1	1	4.532758	215.4066	3.232958
-60	1	1	4.618943	208.8061	2.932828
-40	1	1	4.44342	203.9608	2.5617
-20	1	1	3.97739	200.9975	2.128285
0	1	1	3.332139	200	1.651697
20	1	.9999993	2.540665	200.9975	1.165581
40	.9999931	.908713	1.76834	203.9608	.7394171
60	.9933111	.467368	1.110188	208.8061	.5637183
80	.9998733	.7874625	.9039734	215.4066	.7782836
100	.9999999	.9856169	1.086971	223.6068	1.134662
120	1	.9992399	1.254863	233.2381	1.490748
140	1	.9998478	1.308583	244.1311	1.811734
160	1	.9998431	1.29482	256.125	2.089946
180	1	.9994995	1.230946	269.0725	2.326072
200	1	.9974561	1.131703	282.8427	2.523877
220	1	.9872744	1.017684	297.3214	2.688154
240	.999999	.9510013	.9018685	312.41	2.823785
260	.9999747	.8646194	.7917129	328.0244	2.935307
280	.9996544	.7242736	.6909791	344.093	3.026728
300	.9973592	.5582789	.6011183	360.5551	3.10149
320	.9876337	.4037092	.5222269	377.3593	3.162495
340	.9608865	.2816961	.4536729	394.4617	3.212155
360	.9086407	.1948741	.3944811	411.8252	3.252463
380	.8301703	.1365131	.3443362	429.4182	3.285058
400	.7333605	9.820431E-02	.3014853	447.2136	3.311281
420	.6299387	.0731037	.2645045	465.1881	3.332228
440	.5302704	5.646636E-02	.2325951	483.3218	3.348799
460	.4408869	4.522933E-02	.205043	501.5974	3.361725
480	.3645446	3.747106E-02	.1812233	520	3.371603
500	.3013791	3.199235E-02	.1605954	538.5165	3.378925

Table 3. An example of a lateral range function values printout.

The heading for the crosscorrelation values is p(cc)  
and the heading for the square law values is p(sl).

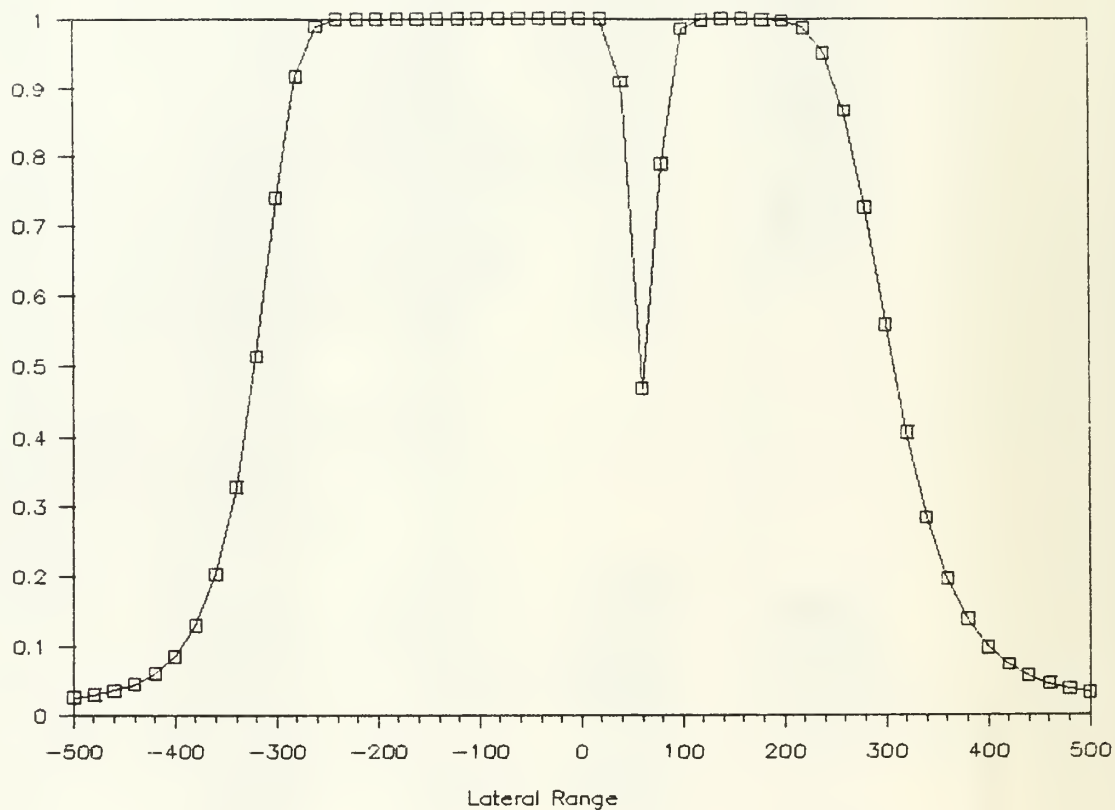


Figure 3. A plot of the square law lateral range function values that are listed in Table 3. The horizontal axis is encounter horizontal range at CPA in meters. The vertical axis is encounter detection probability.

data.mad magnetic signal values  
for a lateral range of 40 meters

relative CPA distance s in meters	magnetic signal Hs in gamma
-927.4282	-1.619177E-02
-881.0568	-1.834315E-02
-834.6854	-2.086144E-02
-788.314	-2.381637E-02
-741.9426	-2.728804E-02
-695.5712	-.0313641
-649.1998	-3.613118E-02
-602.8283	-4.165412E-02
-556.4569	-4.792992E-02
-510.0855	-5.478963E-02
-463.7141	-6.169327E-02
-417.3427	-6.730653E-02
-370.9713	-.0686387
-324.5999	-5.933568E-02
-278.2285	-2.652314E-02
-231.8571	.0540608
-185.4856	.2215785
-139.1142	.5163918
-92.74282	.9104893
-46.37141	1.184586
0	1.003207
46.37141	.3837067
92.74282	-.2252113
139.1142	-.5323231
185.4856	-.5837545
231.8571	-.5168925
278.2285	-.4202569
324.5999	-.3304064
370.9713	-.2572838
417.3427	-.2007385
463.7141	-.1577988
510.0855	-.125285
556.4569	-.1005532
602.8283	-8.158605E-02
649.1998	-6.689598E-02
695.5712	-5.539922E-02
741.9426	-4.630719E-02
788.314	-3.904362E-02
834.6854	-3.318428E-02
881.0568	-.0284142
927.4282	-2.449729E-02

Table 4. An example of a magnetic signal values printout.



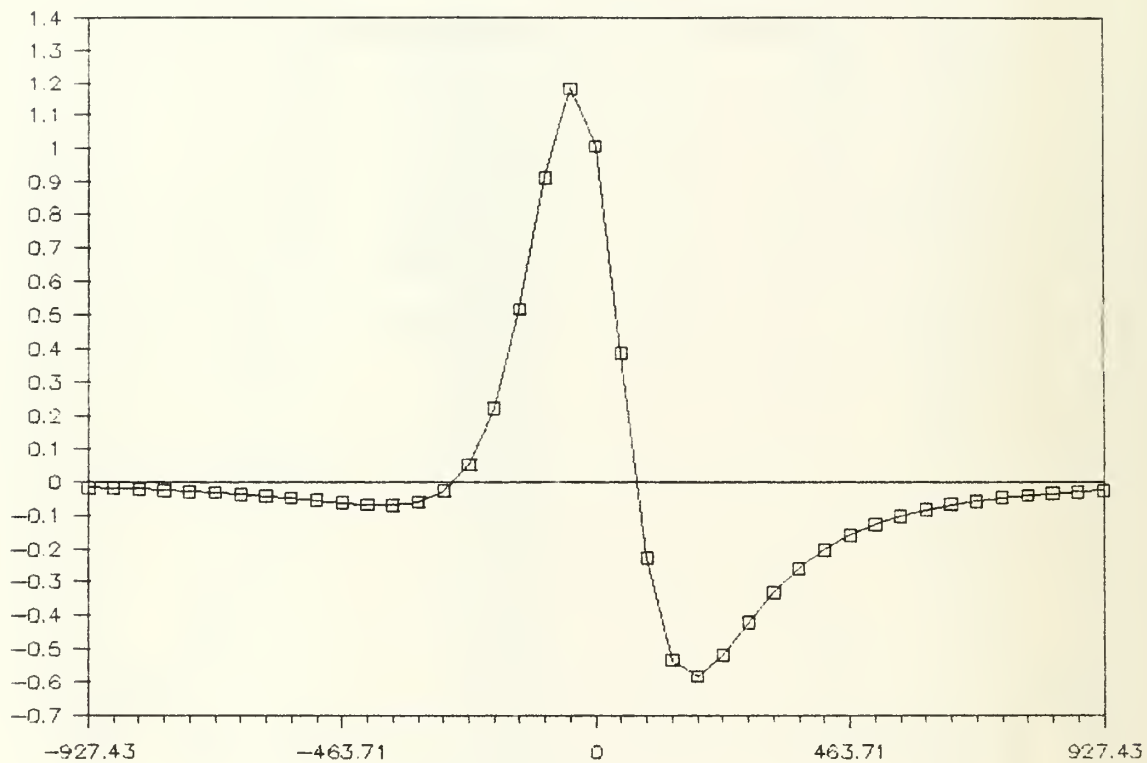


Figure 4. A plot of the magnetic signal values listed in Table 4.  
The horizontal axis is relative CPA distance in meters.  
The vertical axis is magnetic signal in gamma.

## Appendix 10. A Program Listing

```
10 CLS : CLEAR
20 PRINT "Magnetic Anomaly Detection (MAD) Lateral Range Function Program"
30 DIM X0(70), R0(70), H(70), HS(70, 150), K(70), PDCC(70), PDSL(70)
40 PI = 3.141592654#: CON = 2 * PI / 360: KON = 1852 / 3600: N$ = "MAD.BAS"
50 Q0 = .2316419: Q1 = .31938153#: Q2 = -.356563782#: Q3 = 1.781477937#
60 Q4 = -1.821255978#: Q5 = 1.330274429#
70 I1 = 2.515517: I2 = .802853: I3 = .010328: I4 = 1.432788: I5 = .189269: I6
= .001308
80 PRINT
90 INPUT "generate data or print a program data file (g/p)"; A$
100 IF A$ = "G" OR A$ = "g" THEN GOTO 120
110 IF A$ = "P" OR A$ = "p" THEN GOTO 2960 ELSE GOTO 90
120 PRINT : A$ = "a"
130 INPUT "magnetic, processing & kinematic data entry by combined file
(y/n)"; A$
140 IF A$ = "Y" OR A$ = "y" THEN GOTO 2070
150 IF A$ = "N" OR A$ = "n" THEN GOTO 160 ELSE GOTO 130
160 INPUT "magnetic data entry by file/keyboard (f/k)"; A$
170 IF A$ = "F" OR A$ = "f" THEN GOTO 780
180 IF A$ = "K" OR A$ = "k" THEN GOTO 190 ELSE GOTO 160
190 INPUT "latitude in decimal degrees (N +)"; LAT: LATR = LAT * CON
200 INPUT "longitude in decimal degrees (E +)"; LNG: LNGR = -LNG * CON
210 INPUT "variation in decimal degrees (E +)"; DEC: DECR = DEC * CON
220 A$ = "a"
230 INPUT "input dip angle (y/n)"; A$
240 IF A$ = "Y" OR A$ = "y" THEN GOTO 260
250 IF A$ = "N" OR A$ = "n" THEN GOTO 290 ELSE GOTO 230
260 INPUT "dip angle in decimal degrees (north magnetic hemisphere +)"; DIP
270 DIPR = DIP * CON
280 GOTO 390
290 LATPR = 76 * CON: LNGPR = 100 * CON
300 D = SIN(LNGR - LNGPR) * COS(LATR): E = COS(LNGR - LNGPR) * COS(LATR): F =
SIN(LATR)
310 X = E: Y = F
320 GOSUB 4060
330 T = T - (90 * CON - LATPR): E = R * SIN(T): F = R * COS(T)
340 X = E: Y = D
350 GOSUB 4060
360 X = F: Y = R
370 GOSUB 4060
380 DIPR = ATN(2 * TAN(T)): DIP = DIPR / CON
390 A$ = "a"
400 INPUT "input encounter magnetic field intensity (y/n)"; A$
410 IF A$ = "Y" OR A$ = "y" THEN GOTO 430
420 IF A$ = "N" OR A$ = "n" THEN GOTO 450 ELSE GOTO 400
430 INPUT "encounter magnetic field intensity in gamma"; HE
440 GOTO 460
450 HE = 70000! / SQR(3 * COS(DIPR) * COS(DIPR) + 1)
460 PLM = 0: PIM = 0: PVM = 0
470 A$ = "a"
```

```

480 INPUT "input target permanent dipole moments (y/n)"; A$
490 IF A$ = "Y" OR A$ = "y" THEN GOTO 510
500 IF A$ = "N" OR A$ = "n" THEN GOTO 540 ELSE GOTO 480
510 INPUT "permanent longitudinal moment in oersted-cm3 (stern-to-bow +)"; PLM
520 INPUT "permanent transverse moment in oersted-cm3 (port-to-starboard +)";
PIM
530 INPUT "permanent vertical moment in oersted-cm3 (downward +)"; PVM
540 INPUT "target displacement in tons"; WT
550 A$ = "a"
560 INPUT "input target permeability coefficients or factors (c/f)"; A$
570 IF A$ = "C" OR A$ = "c" THEN GOTO 590
580 IF A$ = "F" OR A$ = "f" THEN GOTO 630 ELSE GOTO 560
590 INPUT "longitudinal permeability coefficient in cgs units"; KL
600 INPUT "transverse permeability coefficient in cgs units"; KT
610 INPUT "vertical permeability coefficient in cgs units"; KV
620 FL = KL / WT: FT = KT / WT: FV = KV / WT: GOTO 710
630 INPUT "longitudinal displacement factor in cgs units"; FL
640 INPUT "transverse displacement factor in cgs units"; FT
650 INPUT "vertical displacement factor in cgs units"; FV
660 KL = FL * WT: KT = FT * WT: KV = FV * WT
670 A$ = "a"
680 INPUT "generate a magnetic data file (y/n)"; A$
690 IF A$ = "Y" OR A$ = "y" THEN GOTO 710
700 IF A$ = "N" OR A$ = "n" THEN GOTO 850 ELSE GOTO 680
710 INPUT "magnetic data file name"; M$
720 ON ERROR GOTO 730: GOTO 740
730 RESUME 710
740 OPEN "O", #1, M$
750 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PIM, PVM, WT, KL, KT, KV, FL, FT,
FV
760 CLOSE
770 GOTO 850
780 INPUT "magnetic data file name"; M$
790 ON ERROR GOTO 800: GOTO 810
800 RESUME 780
810 OPEN "I", #1, M$
820 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PIM, PVM, WT, KL, KT, KV, FL, FT,
FV
830 CLOSE
840 DIPR = DIP * CON
850 A$ = "a"
860 INPUT "processing data entry by file/keyboard (f/k)"; A$
870 IF A$ = "F" OR A$ = "f" THEN GOTO 1450
880 IF A$ = "K" OR A$ = "k" THEN GOTO 890 ELSE GOTO 860
890 A$ = "a"
900 INPUT "input sampling period (y/n)"; A$
910 IF A$ = "Y" OR A$ = "y" THEN GOTO 930
920 IF A$ = "N" OR A$ = "n" THEN GOTO 960 ELSE GOTO 900
930 INPUT "sampling period in seconds"; DT
940 IF DT <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
930
950 GOTO 1090

```

```

960 A$ = "a"
970 INPUT "input maximum magnetic signal frequency (y/n)"; A$
980 IF A$ = "Y" OR A$ = "y" THEN GOTO 1000
990 IF A$ = "N" OR A$ = "n" THEN GOTO 1030 ELSE GOTO 970
1000 INPUT "maximum magnetic signal frequency in Hertz"; MAXF
1010 IF MAXF <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
1000
1020 GOTO 1080
1030 INPUT "minimum target slant range at CPA in meters"; MINRO
1040 IF MINRO <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT :
GOTO 1030
1050 INPUT "maximum magnetometer relative speed in knots"; MAXVMK
1060 MAXVM = MAXVMK * KON: MAXF = 2 * MAXVM / MINRO
1070 IF MAXVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT :
GOTO 1050
1080 DT = 1 / (2 * MAXF): REM low pass filter, Nyquist sampling rate
1090 A$ = "a"
1100 INPUT "input integration time (y/n)"; A$
1110 IF A$ = "Y" OR A$ = "y" THEN GOTO 1130
1120 IF A$ = "N" OR A$ = "n" THEN GOTO 1210 ELSE GOTO 1100
1130 INPUT "integration time in seconds"; IT
1140 IF IT >= DT THEN GOTO 1170
1150 PRINT : PRINT "IT = " + STR$(IT) + " seconds - minimum = " + STR$(DT) + "
seconds": PRINT
1160 GOTO 1130
1170 NS = 2 * INT(IT / DT / 2) + 1: REM adj number of samples per integration
time
1180 IF NS <= 151 THEN GOTO 1340
1190 PRINT : PRINT "IT = " + STR$(IT) + " seconds - maximum = " + STR$(150 *
DT) + " seconds": PRINT
1200 GOTO 1130
1210 INPUT "maximum target slant range at CPA in meters"; MAXRO
1220 IF MAXRO <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT :
GOTO 1210
1230 INPUT "minimum magnetometer relative speed in knots"; MINVMK
1240 MINVM = MINVMK * KON
1250 IF MINVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT :
GOTO 1230
1260 IT = 2 * MAXRO / MINVM
1270 IF IT >= DT THEN GOTO 1300
1280 PRINT : PRINT "IT = " + STR$(IT) + " seconds - minimum = " + STR$(DT) + "
seconds": PRINT
1290 GOTO 1100
1300 NS = 2 * INT(IT / DT / 2) + 1: REM adjusted number of samples per
integration time
1310 IF NS <= 151 THEN GOTO 1340
1320 PRINT : PRINT "IT = " + STR$(IT) + " seconds - minimum = " + STR$(150 *
DT) + " seconds": PRINT
1330 GOTO 1100
1340 A$ = "a"
1350 INPUT "generate a processing data file (y/n)"; A$
1360 IF A$ = "Y" OR A$ = "y" THEN GOTO 1380

```



```

1370 IF A$ = "N" OR A$ = "n" THEN GOTO 1510 ELSE GOTO 1350
1380 INPUT "processing data file name"; P$
1390 ON ERROR GOTO 1400: GOTO 1410
1400 RESUME 1380
1410 OPEN "O", #1, P$
1420 WRITE #1, DT, IT, NS
1430 CLOSE
1440 GOTO 1510
1450 INPUT "processing data file name"; P$
1460 ON ERROR GOTO 1470: GOTO 1480
1470 RESUME 1450
1480 OPEN "I", #1, P$
1490 INPUT #1, DT, IT, NS
1500 CLOSE
1510 A$ = "a"
1520 INPUT "kinematic data entry by file/keyboard (f/k)"; A$
1530 IF A$ = "F" OR A$ = "f" THEN GOTO 1890
1540 IF A$ = "K" OR A$ = "k" THEN GOTO 1550 ELSE GOTO 1510
1550 INPUT "magnetometer course in decimal degrees (0 if at rest)"; CM
1560 MCM = (CM - DEC): MCMR = MCM * CON: REM magnetometer magnetic course
1570 INPUT "magnetometer speed in knots"; VMK
1580 INPUT "magnetometer altitude in meters (below 0 is -)"; AM
1590 INPUT "target course in decimal degrees"; CT
1600 MCT = (CT - DEC): MCTR = MCT * CON: REM target magnetic course
1610 INPUT "target speed in knots"; VTK
1620 INPUT "target depth in meters (above 0 is -)"; AT
1630 ILM = KL * HE * COS(DIPR) * COS(MCTR): ITM = -KT * HE * COS(DIPR) *
SIN(MCTR)
1640 IVM = KV * HE * SIN(DIPR)
1650 DMX = (PLM + ILM) * SIN(MCTR) + (PIM + ITM) * COS(MCTR)
1660 DMY = (PLM + ILM) * COS(MCTR) - (PIM + ITM) * SIN(MCTR)
1670 DMV = PVM + IVM
1680 X = DMX: Y = DMY
1690 GOSUB 4060
1700 OMLR = T: X = DMV: Y = R
1710 GOSUB 4060
1720 DM = R: OMR = T
1730 WVK = VMK * SIN(MCMR) - VTK * SIN(MCTR): WYK = VMK * COS(MCMR) - VTK *
COS(MCTR)
1740 Z = AM + AT: REM vertical separation (- for magnetometer below target)
1750 X = WVK: Y = WYK: GOSUB 4060
1760 CR = T: WK = R: REM relative magnetometer magnetic course and speed
1770 OML = OMLR / CON: OM = OMR / CON: C = CR / CON
1780 A$ = "a"
1790 INPUT "generate a kinematic data file (y/n)"; A$
1800 IF A$ = "Y" OR A$ = "y" THEN GOTO 1820
1810 IF A$ = "N" OR A$ = "n" THEN GOTO 1950 ELSE GOTO 1790
1820 INPUT "kinematic data file name"; K$
1830 ON ERROR GOTO 1840: GOTO 1850
1840 RESUME 1820
1850 OPEN "O", #1, K$
1860 WRITE #1, CM, VMK, AM, CT, VTK, AT, Z, C, WK, ILM, ITM, IVM, DM, OML, OM

```

```

1870 CLOSE
1880 GOTO 1950
1890 INPUT "kinematic data file name"; K$
1900 ON ERROR GOTO 1910: GOTO 1920
1910 RESUME 1890
1920 OPEN "I", #1, K$
1930 INPUT #1, CM, VMK, AM, CT, VTK, AT, Z, C, WK, ILM, ITM, IVM, DM, OML, OM
1940 CLOSE
1950 A$ = "a"
1960 INPUT "generate a combined magnetic, processing & kinematic data file
(y/n)"; A$
1970 IF A$ = "Y" OR A$ = "y" THEN GOTO 1990
1980 IF A$ = "N" OR A$ = "n" THEN GOTO 2140 ELSE GOTO 1960
1990 INPUT "combined magnetic, processing & kinematic data file name"; E$
2000 ON ERROR GOTO 2010: GOTO 2020
2010 RESUME 1990
2020 OPEN "O", #1, E$
2030 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT,
FV, DT, IT
2040 WRITE #1, NS, CM, VMK, AM, CT, VTK, AT, Z, C, WK, ILM, ITM, IVM, DM, OML,
OM, M$, P$, K$
2050 CLOSE
2060 GOTO 2140
2070 INPUT "combined magnetic, processing & kinematic data file name"; E$
2080 ON ERROR GOTO 2090: GOTO 2100
2090 RESUME 2070
2100 OPEN "I", #1, E$
2110 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT,
FV, DT, IT
2120 INPUT #1, NS, CM, VMK, AM, CT, VTK, AT, Z, C, WK, ILM, ITM, IVM, DM, OML,
OM, M$, P$, K$
2130 CLOSE
2140 OMLR = OML * CON: OMR = OM * CON: DIPR = DIP * CON
2150 CR = C * CON: W = WK * CON
2160 INPUT "required false alarm rate in false alarms per hour"; FAR
2170 PF = FAR * IT / 3600: REM false alarm probability
2180 Y = PF: IF PF > .5 THEN Y = 1 - Y: REM inverse normal approximation
2190 Y = SQR(LOG(1 / Y / Y))
2200 Y = Y - (I1 + Y * (I2 + I3 * Y)) / (1 + Y * (I4 + Y * (I5 + I6 * Y)))
2210 IF PF < .5 THEN Y = -Y
2220 ZP = -Y
2230 CHI = NS * (1 - 2 / 9 / NS + ZP * SQR(2 / 9 / NS)) ^ 3: REM inverse
chi-square approximation
2240 INPUT "magnetic noise in gamma"; SIG
2250 INPUT "maximum lateral range in meters"; LRM
2260 INPUT "lateral range step in meters"; ST
2270 IF ST <= LRM THEN GOTO 2290
2280 PRINT : PRINT "maximum step is " + STR$(LRM) + " meters": PRINT : GOTO
2260
2290 NC = 2 * INT(LRM / ST) + 1: REM number of lateral range function values
2300 IF NC <= 71 THEN GOTO 2320

```

```

2310 PRINT : PRINT "minimum step is " + STR$(LRM / 35) + " meters": PRINT :
GOTO 2260
2320 ATT = DT * NS: REM adjusted integration time
2330 DS = W * DT: REM distance between samples on the relative track
2340 X0 = -(NC - 1) / 2 * ST
2350 FOR I = 0 TO NC - 1
2360 X0(I) = X0
2370 X = X0: Y = Z
2380 GOSUB 4060
2390 R0 = R: R0(I) = R: REM target slant range at CPA in meters
2400 DELR = T: REM target depression angle complement at CPA in radians
2410 IF R0 = 0 THEN GOTO 2730: REM zero lateral range and vertical separation
2420 DMF = DM / 10 / R0 ^ 3: REM dipole moment factor
2430 L = COS(OMR) * COS(CR - OMLR)
2440 M = COS(DELR) * COS(OMR) * SIN(CR - OMLR) - SIN(DELR) * SIN(OMR)
2450 N = -SIN(DELR) * COS(OMR) * SIN(CR - OMLR) - COS(DELR) * SIN(OMR)
2460 L1 = COS(DIPR) * COS(CR)
2470 M1 = COS(DELR) * COS(DIPR) * SIN(CR) - SIN(DELR) * SIN(DIPR)
2480 N1 = -SIN(DELR) * COS(DIPR) * SIN(CR) - COS(DELR) * SIN(DIPR)
2490 A2 = 2 * L * L1 - M * M1 - N * N1: REM Anderson Function Coefficient
2500 A1 = 3 * (N * L1 + L * N1): REM Anderson Function Coefficient
2510 A0 = 2 * N * N1 - L * L1 - M * M1: REM Anderson Function Coefficient
2520 SUM = 0: HMAX = 0: HMIN = 0
2530 FOR J = 0 TO NS - 1
2540 S = (J - (NS - 1) / 2) * DS: BA = S / R0: REM Anderson Function Argument
2550 AF = 1 / (1 + BA * BA) ^ 2.5: REM Anderson Function Factor
2560 HSF = (A2 * BA * BA + A1 * BA + A0) * AF: REM magnetic signal factor
2570 HS(I, J) = DMF * HSF: REM magnetic signal value
2580 IF HS(I, J) > HMAX THEN HMAX = HS(I, J)
2590 IF HS(I, J) < HMIN THEN HMIN = HS(I, J)
2600 SUM = SUM + HSF * HSF
2610 NEXT J
2620 H(I) = HMAX - HMIN
2630 K(I) = SQR(SUM)
2640 VV = -ZP + DMF * SQR(SUM) / SIG
2650 LAM = DMF * DMF * SUM / (SIG * SIG): AA = NS + LAM: BB = 1 + LAM / (NS + LAM)
2660 ZN = -SQR(2 * CHI / BB) + SQR(2 * AA / BB - 1): X1 = VV
2670 GOSUB 4120
2680 IF Y1 > 1 THEN Y1 = 1
2690 PDCC(I) = Y1: X1 = ZN
2700 GOSUB 4120
2710 IF Y1 > 1 THEN Y1 = 1
2720 PDSL(I) = Y1
2730 X0 = X0 + ST
2740 NEXT I
2750 A$ = "a"
2760 INPUT "generate a program data file (y/n)"; A$
2770 IF A$ = "Y" OR A$ = "y" THEN GOTO 2790
2780 IF A$ = "N" OR A$ = "n" THEN GOTO 3120 ELSE GOTO 2760
2790 INPUT "program data file name"; D$
2800 ON ERROR GOTO 2810: GOTO 2820

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2810 RESUME 2790
2820 OPEN "O", #1, D$
2830 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT,
FV, DT, IT, AIT
2840 WRITE #1, NS, CM, VM, AM, CT, VT, AT, ILM, ITM, IVM, Z, C, W, DM, OML,
OM, FAR, PF, SIG, ST
2850 WRITE #1, LRM, DS, NC, M$, P$, K$, E$
2860 FOR I = 0 TO NC - 1
2870 WRITE #1, XO(I), PDCC(I), PDSL(I), K(I), H(I), RO(I)
2880 NEXT I
2890 FOR I = 0 TO NC - 1
2900 FOR J = 0 TO NS - 1
2910 WRITE #1, HS(I, J)
2920 NEXT J
2930 NEXT I
2940 CLOSE
2950 GOTO 3120
2960 INPUT "program data file name"; D$
2970 ON ERROR GOTO 2980: GOTO 2990
2980 RESUME 2960
2990 OPEN "I", #1, D$
3000 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT,
FV, DT, IT, AIT
3010 INPUT #1, NS, CM, VM, AM, CT, VT, AT, ILM, ITM, IVM, Z, C, W, DM, OML,
OM, FAR, PF, SIG, ST
3020 INPUT #1, LRM, DS, NC, M$, P$, K$, E$
3030 FOR I = 0 TO NC - 1
3040 INPUT #1, XO(I), PDCC(I), PDSL(I), K(I), H(I), RO(I)
3050 NEXT I
3060 FOR I = 0 TO NC - 1
3070 FOR J = 0 TO NS - 1
3080 INPUT #1, HS(I, J)
3090 NEXT J
3100 NEXT I
3110 CLOSE
3120 PRINT : A$ = "a"
3130 INPUT "print encounter parameter values (y/n)"; A$
3140 IF A$ = "Y" OR A$ = "y" THEN GOTO 3160
3150 IF A$ = "N" OR A$ = "n" THEN GOTO 3670 ELSE GOTO 3130
3160 LPRINT
3170 LPRINT "program file name" " + N$
3180 LPRINT "program data file name" " + D$
3190 LPRINT "magnetic data file name" " + M$
3200 LPRINT "processing data file name" " + P$
3210 LPRINT "kinematic data file name" " + K$
3220 LPRINT "combined magnetic, processing & kinematic data file name" " + E$
3230 LPRINT "encounter latitude (decimal degrees)" "; SPC(2);
LAT
3240 LPRINT "encounter longitude (decimal degrees)" "; SPC(2);
LNG
3250 LPRINT "encounter variation (decimal degrees)" "; SPC(2);
DEC

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3260 LPRINT "encounter dip angle (decimal degrees)           "; SPC(2);
DIP
3270 LPRINT "encounter magnetic field intensity (gamma)      "; SPC(2);
HE
3280 LPRINT "target permanent longitudinal moment (oersted-cm3) "; SPC(2);
PLM
3290 LPRINT "target permanent transverse moment (oersted-cm3)  "; SPC(2);
PIM
3300 LPRINT "target permanent vertical moment (oersted-cm3)     "; SPC(2);
PVM
3310 LPRINT "target displacement (tons)                               "; SPC(2);
WT
3320 LPRINT "target longitudinal permeability coefficient            "; SPC(2);
KL
3330 LPRINT "target transverse permeability coefficient                "; SPC(2);
KT
3340 LPRINT "target vertical permeability coefficient                  "; SPC(2);
KV
3350 LPRINT "target longitudinal permeability factor                    "; SPC(2);
FL
3360 LPRINT "target transverse permeability factor                      "; SPC(2);
FT
3370 LPRINT "target vertical permeability factor                        "; SPC(2);
FV
3380 LPRINT "sampling period (seconds)                                "; SPC(2);
DT
3390 LPRINT "integration time (seconds)                                "; SPC(2);
IT
3400 LPRINT "adjusted integration time (seconds)                        "; SPC(2);
AIT
3410 LPRINT "number of samples per encounter                            "; SPC(2);
NS
3420 LPRINT "magnetometer course (decimal degrees)                      "; SPC(2);
CM
3430 LPRINT "magnetometer speed (knots)                                 "; SPC(2);
VMK
3440 LPRINT "magnetometer altitude (meters)                             "; SPC(2);
AM
3450 LPRINT "target course (decimal degrees)                            "; SPC(2);
CT
3460 LPRINT "target speed (knots)                                        "; SPC(2);
VTK
3470 LPRINT "target depth (meters)                                       "; SPC(2);
AT
3480 LPRINT "magnetometer relative magnetic course (decimal degrees)"; SPC(2);
C
3490 LPRINT "magnetometer relative speed (knots)                        "; SPC(2);
WK
3500 LPRINT "magnetometer-target vertical separation (meters)           "; SPC(2);
Z
3510 LPRINT "target induced longitudinal dipole moment (oersted-cm3)"; SPC(2);
ILM

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3520 LPRINT "target induced transverse dipole moment (oersted-cm3) "; SPC(2);
TIM
3530 LPRINT "target induced vertical dipole moment (oersted-cm3) "; SPC(2);
IVM
3540 LPRINT "target dipole moment (oersted-cm3) "; SPC(2);
DM
3550 LPRINT "target dipole moment azimuth (decimal degrees) "; SPC(2);
OML
3560 LPRINT "target dipole moment depression (decimal degrees) "; SPC(2);
OM
3570 LPRINT "distance between samples on the relative track (meters)"; SPC(2);
DS
3580 LPRINT "false alarm rate (false alarms per hour) "; SPC(2);
FAR
3590 LPRINT "false alarm probability "; SPC(2);
PF
3600 LPRINT "magnetic noise (gamma) "; SPC(2);
SIG
3610 LPRINT "maximum lateral range (meters) "; SPC(2);
LRM
3620 LPRINT "lateral range step (meters) "; SPC(2);
ST
3630 LPRINT "number of lateral range function values "; SPC(2);
NC
3640 FOR I = 0 TO 15
3650 LPRINT
3660 NEXT I
3670 PRINT : A$ = "a"
3680 INPUT "print lateral range function values (y/n)"; A$
3690 IF A$ = "Y" OR A$ = "y" THEN GOTO 3710
3700 IF A$ = "N" OR A$ = "n" THEN GOTO 3790 ELSE GOTO 3680
3710 LPRINT D$; " lateral range function values"
3720 LPRINT : LPRINT
3730 LPRINT "L          p(cc)          p(sl)          H          R
k"
3740 LPRINT "meters          gamma          meters"
3750 LPRINT
3760 FOR I = 0 TO NC - 1
3770 LPRINT X0(I); TAB(10); PDCC(I); TAB(24); PDSL(I); TAB(38); H(I); TAB(52);
R0(I); TAB(70); K(I)
3780 NEXT I
3790 PRINT : A$ = "a"
3800 INPUT "print magnetic signal values (y/n)"; A$
3810 IF A$ = "Y" OR A$ = "y" THEN GOTO 3830
3820 IF A$ = "N" OR A$ = "n" THEN GOTO 4010 ELSE GOTO 3800
3830 PRINT : PRINT "identify signal by encounter lateral range index"
3840 PRINT "lateral range equals the index times " + STR$(ST) + " meters"
3850 PRINT "index values: -" + STR$((NC - 1) / 2) + " to " + STR$((NC - 1) /
2)
3860 INPUT "lateral range index"; K: I = K + (NC - 1) / 2
3870 LPRINT : LPRINT : LPRINT
3880 LPRINT D$; " magnetic signal values"

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3890 LPRINT "for a lateral range of "; K * ST; " meters"
3900 LPRINT
3910 LPRINT "relative CPA distance"; TAB(35); "magnetic signal"
3920 LPRINT "s in meters"; TAB(35); "Hs in gamma"
3930 LPRINT
3940 FOR J = 0 TO NS - 1
3950 LPRINT (J - (NS - 1) / 2) * DS; TAB(35); HS(I, J)
3960 NEXT J
3970 PRINT : A$ = "a"
3980 INPUT "print magnetic signal values for a different lateral range (y/n) "
A$
3990 IF A$ = "Y" OR A$ = "y" THEN GOTO 3830
4000 IF A$ = "N" OR A$ = "n" THEN GOTO 4010 ELSE GOTO 3980
4010 PRINT : A$ = "a"
4020 INPUT "continue to use the program (y/n)"; A$
4030 IF A$ = "Y" OR A$ = "y" THEN GOTO 10
4040 IF A$ = "N" OR A$ = "n" THEN GOTO 4050 ELSE GOTO 4020
4050 END
4060 R = SQR(X * X + Y * Y): REM rectangular to polar conversion
4070 IF R = 0 THEN T = 0: RETURN
4080 IF ABS(X / R) = 1 THEN Q = SGN(X) * (PI / 2) ELSE Q = ATN(X / R / SQR(1
X * X / R / R))
4090 IF ABS(Y / R) = 1 THEN T = (PI / 2) * (1 - SGN(Y)) ELSE T = (PI / 2) -
ATN(Y / R / SQR(1 - Y * Y / R / R))
4100 IF Q < 0 THEN T = 2 * PI - T
4110 RETURN
4120 Y1 = X1: IF X1 < 0 THEN Y1 = -Y1: REM normal approximation
4130 G = 1 / (1 + Q0 * Y1)
4140 Y1 = EXP(-Y1 * Y1 / 2) / SQR(2 * PI) * G * (Q1 + G * (Q2 + G * (Q3 + G *
(Q4 + G * Q5))))
4150 IF X1 >= 0 THEN Y1 = 1 - Y1
4160 RETURN

```

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